COMPACT ZONAL CAVITY CALCULATIONS

BY

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The typical light fixture data sheet has a table of 150 numbers that represent the coefficients of utilization. These figures must be interpreted three ways to generate the Room Cavity Ratio. The RCR is used to calculate the ambient light level, in foot-candles, by the Zonal Cavity Method. This report describes a mathematical technique to reduce the 150 numbers of the table to eight. The resultant function performs the three way interpolation automatically and computes values within a few percent of the tabulated numbers. The mathematics used to assimilate the function are simple and direct. A significant application of the eight-variable function is in computer programming. Calculation can be performed faster, more accurately and will require less storage capacity.

INTRODUCTION

A typical fluorescent light fixture is represented photometrically by an 11 x 14 matrix of coefficients. The designer must interpolate three ways before reaching the value needed in the zonal cavity calculation, the Room Cavity Ratio (RCR). This is a tedious procedure, so the wall, ceiling and floor reflectances are most often rounded to the nearest ten (e.g. 50, 80, 30) and the RCR is extracted from the table.

A less than exact calculation procedure such as this produces a great inaccuracy in the determination of light levels. The results of this guesswork are inappropriate light levels, excess energy usage and inaccurate specification of fixtures, lamps and luminaire spacing. This report presents a method to reduce the matrix of coefficients with a single function into which the wall, ceiling and floor reflectances are input directly.

A presentation of luminaire photometrics as a single equation will not only facilitate the calculation, but product a more accurate result.

EXPONENTIAL CURVE FITTING

The first task in the process of reducing the data matrix to representative functions was to find the best curve fit for the manufacturer's Room Coefficient Ratio (RCR) values. Several sets of these coefficients were evaluated using linear, exponential, logarithmic and power curve fitting routines. The exponential function was the best fit.

$$CofU - B * EXP(M * RCR) \qquad (1)$$

Where B and M are variables determined by the numerical analysis of the data points. A third variable is also generated, the regression coefficient, R. This number indicate the degree to which the curve approximates the data, with R = 1.0 a perfect fit.

The exponential fit was determined for the set of eleven RCRs for each ceiling/wall reflectance combination in the table of coefficients of a typical fluorescent fixture. The M and B variables are given in Table I. Note the regression coefficients are close to 0.999, which indicates a very good fit. Each of these equations gives the CofUs as a function of RCR as given in Equation 1.

LINEAR APPROXIMATION

The next part of the process is to determine the function of the slope and intercepts for each ceiling reflectance block of values. The four curve fitting routines were tried for these relations and the linear, least-squares fit method gave the highest regression coefficient. This equation is of the form:

$$Y = M * WALL + B \qquad (2)$$

Where M and B are variables calculated by the numerical analysis and Y is either the slope of intercept sought. All these analyses were performed successfully and are described in Table I.

THE WHOLE MATRIX FUNCTION

The last step of the numerical reduction process is to fit the ceiling reflectance into the function by determining how the slopes and intercepts vary as a function of ceiling reflectance (e.g. holding the wall reflectance constant). This is simpler to follow if the function to this point is given:

$$CofU = [Bm * WALL + Bb] * EXP \{[Mm * WALL + Mb] * RCR\} \qquad (3)$$

This is the original exponential equation with the linear approximations proven valid in the previous section substituted for the variables M and B in equation 1. This expression is a function of RCR and wall reflectance, and the remaining variables can now be found as functions of the ceiling reflectance.

These final equations are given in Table I, where the subscripts of B and M indicate the slope and intercept. Note that all the relations were aptly represented by linear functions, with the exception of Mb, which was better fit by a logarithmic curve.

Substituting these functions into equation 3, there results a function of eight variables. Table II gives the values produced by this new function, and they are no where more than one point (+/- 2%) away from the manufacturer's data. This is an exemplary fit.

TABLE I

CEILING	WALL	M	B	R		M	B	R
80	70	-.0803	98.65	.999	M	-.0009955	.1540	.983
	50	-.1080	97.35	.999	B	.08955	9263	.992
	30	-.1287	95.58	.999				
	10	-.1398	93.27	.996				
70	70	-.0814	96.48	.999	M	-.0009915	.1539	.992
	50	-.1077	95.47	.999	B	.07545	91.38	.994
	30	-.1260	93.55	.999				
	10	-.1414	92.09	.998				
50	50	-.1059	91.72	.999	M	-.0008925	.1513	.998
	30	-.1260	90.90	.999	B	.04450	89.52	.999
	10	-.1416	89.94	.999				
30	50	-.1043	87.79	.999	M	-.0008675	.1427	.999
	30	-.1217	87.11	.999	B	.02600	86.44	.985
	10	-.1379	86.75	.999				

$CofU = Be^{M(RCR)}$ → $Y = Mx + B$ →

	M	S	CURVE	R
Mm	28.76	7714	S × (-1/1000)	.961
Mb	.006729	.1245	L	.994
Bm	129.4	-1555	S × (1/10,000)	.994
Bb	.1206	83.06	S	.994

Mm = -(28.6 (CLG) + 7714)/1E7
Mb = .1245 + .006729 ln (CLG)
Bm = (129.4 (CLG) -1555)/1E5
Bb = (.1206 (CLG) +83.06)

where CofU = (Bm(WALL) + Bb) EXP[(Mm (WALL) + Mb)RCR]